

Abstracts

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ON CONVERGENCE ALMOST EVERYWHERE OF SPHERICAL PARTIAL SUMS

Ravshan Ashurov

Institute of Advanced Technology (ITMA), Universiti Putra Malaysia

ashurovr@yahoo.com

Coauthors: n/a

ON CONVERGENCE ALMOST EVERYWHERE OF SPHERICAL PARTIAL SUMS Ravshan Ashurov

Institute of Advanced Technology (ITMA), Universiti Putra Malaysia 43400 UPM Serdang, Selangor Darul Ehsan, MALAYSIA.

ashurovr@yahoo.com,

Abstract

We investigate convergence almost everywhere of spherical partial sums of the multiple Fourier series (define by $S_\lambda f$) on sets from outside of support of a given function. Following Ilin we say that the generalized localization principle for $S_\lambda f$ holds in L_p , if for any function $f \in L_p(T^N)$ spherical partial sums tend to zero almost-everywhere on $T^N \setminus \text{supp} f$. In this talk we give a complete solution of the problem of generalized localization for spherical partial sums of multiple Fourier series in L_p spaces. It was known that if $p < 2$, then the generalized localization for the spherical partial sums is not valid in L_p . So we prove that if $p \geq 2$, then there is the generalized localization principle in L_p spaces.

Keywords: Multiple Fourier series, spherical partial sums, convergence almost-everywhere, generalized localization.

Two-Player Evasion-Interrogation Games with Uncertainty

H.T. Banks

Center for Research in Scientific Computation North Carolina State University

htbanks@ncsu.edu

We first consider electromagnetic interrogation in the context of evader-interrogator games where each player has uncertain information about the adversary's capabilities. The mathematical formulation results in a static minmax problem over spaces of probability measures. The minmax cost functional is based on reflected fields from an object such as an airfoil or missile. To investigate theoretical, computational and approximation issues for these problems, it is necessary to put a topology on the space of probability measures. A natural choice for this is the Prohorov metric topology. One can then develop well-posedness and approximation results for the associated minmax problems. Efficient computational methods that correspond to Von Neumann's *finite mixed strategies* can readily be developed in this context.

Our efforts on *static* two person non-cooperative games demonstrate the feasibility and the potential usefulness of developing counter-interrogation (CI) and counter-counter-interrogation (CCI) theories for problems with uncertainty. We

will also present our initial efforts on a conceptual, theoretical, and computational framework for CI/CCI *dynamic games* with uncertainty. This eventually involves allowing both the evader and interrogator to make dynamic changes to their strategies while each has only partial information (stochastic in nature) of the others changes. We will describe a number of different formulations among possible stochastic approaches. The framework we describe makes essential use of much of the important theoretical and computational results involving feedback control for partial differential equations as developed during the past several decades. We illustrate some of the technical ideas in the context of an evaders formulation to thwart detection in the presence of dynamic interrogation with uncertainty.

Existence of Solutions for Singular Functional Differential Equations with Upper and Lower Solutions

Daniel C. Biles

Dept. of Math. & CS, Belmont University, 1900 Belmont Blvd., Nashville, TN 37212

`daniel.biles@belmont.edu`

Coauthors: John S. Spraker, Western Kentucky University

We prove a result for existence of solutions for singular functional differential equations using the upper and lower solutions method. A counterexample to weakening the hypotheses is given, and an illustrative example is also included.

Dark optical solitons with power law nonlinearity and time-dependent coefficients

Anjan Biswas

Delaware State University

`biswas.anjan@gmail.com`

Coauthors: Manirupa Saha & Amarendra K. Sarma

This talk is about the dynamics of dark optical solitons that are governed by the nonlinear Schrodingers equation with power law nonlinearity. The solitons are considered in presence of linear attenuation, third order dispersion and self-steepening terms, all with time dependent coefficients. The solitary wave ansatz is used to carry out the integration and an exact soliton solution is obtained. It is only necessary that these time-dependent coefficients be Riemann integrable.

Solving High Rank Lyapunov Equations with Model Reduction

Jeff Borggaard

Interdisciplinary Center for Applied Mathematics, Virginia Tech

jborggaard@vt.edu

Coauthors: Miroslav Stoyanov

The typical technique for solving linear quadratic control problems for partial differential equations is to apply model reduction methods then solve a low dimensional Riccati equation. In this work, we consider the use of model reduction at the end of the process. Namely, we use standard approximation theory for the control problem, then use model reduction methods to solve the high rank Riccati equations. Numerical examples include feedback stabilization of von Karman vortex shedding behind a cylinder by rotation.

Existence and Nonexistence of Positive Solutions For a Class of Superlinear Semipositone Systems

Maya Chhetri

UNC Greensboro

maya@uncg.edu

Coauthors: Petr Girg

In this talk we will consider an elliptic system of the form

$$\left. \begin{aligned} -\Delta u &= \lambda f(v) & \text{in } \Omega \\ -\Delta v &= \lambda g(u) & \text{in } \Omega \\ u = 0 &= v & \text{on } \partial\Omega, \end{aligned} \right\}$$

where $\lambda > 0$ is a parameter and Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$. The nonlinearities $f, g : [0, \infty) \rightarrow R$ are locally Holder continuous functions that are superlinear at infinity and satisfy $f(0) < 0$ and $g(0) < 0$. We will discuss the existence of positive solution for λ small and nonexistence of positive solution for λ large.

Strongly damped wave equation with integral boundary conditions

Jaydev Dabas

Department of Mathematics MNNIT Allahabad-211004, India

jay.dabas@gmail.com

Coauthors: Dharendra Bahuguna (Department of Mathematics I.I.T Kanpur India)

In this work we consider a strongly damped wave equation with integral boundary conditions. We apply the method of semidiscretization in time, also known as the method of lines, to establish the existence and uniqueness of weak solutions.

Materials for Analyzing Sensitivity using the Partial Rank Correlation Coefficient

Jeff Denny

Mercer University

dennyjk@mercer.edu

Coauthors : EmilyPiscitello

The partial rank correlation coefficient (PRCC) gives a measure of the sensitivity of a differential equations model to its parameters. We will present software and materials for introducing undergraduate students to the use of Latin hypercube sampling to randomly sample a parameter space and then use the resulting information to compute the PRCC. Examples will be provided to demonstrate the use of these methods. Students should then be able to apply these techniques to their own modeling projects.

Water Conservation in Georgia A Mathematical Study

Joshua Du

Kennesaw State University

jdu@kennesaw.edu

Coauthors: Kelsey Seufert & Kellyre Gilbert

Water Conservation in Georgia A Mathematical Study By Kelsey Seufert & Kellyre Gilbert

Advisors: Joshua Du & Andrew Whittle Department of Mathematics & Statistics Kennesaw State University Advisors: John Seufert & John Hicks Members of Scientific Advisory Board of Lake Allatoona Preservation Authority Sponsored by the Mentor-Protg Program, College of Science & Mathematics, KSU

Abstract In this project, students collected meteorological data set about Lake Allatoona in the summer & fall of 2008 from a family

lab; studied the correlation of water level, rainfall, temperature, and humidity; and established a mathematical model: a Linear System of First Order Differential Equations. Primary mathematical study shows that the system is stable; the most significant factor to the water level is precipitation; they presented their findings & recommendation in the Cobb County Water Authority about water conservation in Georgia.

A Model of Cell Growth and Reproduction

Sean Ellermeyer

Kennesaw State University

sellerme@kennesaw.edu

Coauthors: Sergei Pilyugin

We consider a size-structured cell population model in which the rate of cell growth is both size and time dependent and the average per capita reproduction rate is specified as a model parameter. It is shown that the model admits classical solutions and the population level and distribution level behaviors of these solutions is determined in terms of the model parameters. The distribution level behavior is found to be different from that found in similar models of cell population dynamics. In particular, rather than convergence to a stable size distribution, we find that size distributions repeat in cycles. This phenomenon is observed in similar models only under special assumptions on the functional form of the size-dependent growth rate factor. All of our main results are illustrated in examples.

Approximation for LQR Control of a Linear Delay Equation

Richard Fabiano

Department of Mathematics and Statistics, University of North Carolina at Greensboro

fabiano@uncg.edu

We consider the LQR problem of minimizing a quadratic cost functional with dynamics governed by the scalar neutral delay equation

$$\frac{d}{dt}[z(t) + cz(t-r)] = az(t) + bz(t-r) + u(t), \quad (1)$$

where $0 < r$, a , b and c are scalars, and $u(t)$ represents a control input. We investigate how changing the norm on the state space affects the behavior of finite dimensional semidiscrete approximation schemes for this problem. Some numerical calculations involving approximations of the feedback gain are given.

On the existence of a double S-shaped bifurcation curve
 Jerome Goddard II
Mississippi State University
 jg440@msstate.edu
 Coauthors: E. Lee, R. Shivaji

We study the positive solutions to boundary value problems of the form

$$\begin{aligned}
 -u'' - \frac{n-1}{r}u' &= \lambda f(u); & \Omega \\
 -\alpha(x,u)u'(r) + [1 - \alpha(x,u)]u(r) &= 0; & |x| = R_1 \\
 \alpha(x,u)u'(r) + [1 - \alpha(x,u)]u(r) &= 0; & |x| = R_2
 \end{aligned}$$

where $\Omega = \{x | R_1 < |x| < R_2\}$ is an annulus in \mathbb{R}^n with $n \geq 1$, λ is a positive parameter, $f : [0, \infty) \rightarrow (0, \infty)$ is a smooth function which is sublinear at ∞ , and $\alpha(x, u) : \Omega \times \mathbb{R} \rightarrow [0, 1]$ is a non-decreasing smooth function. In particular, we discuss the existence of at least two positive radial solutions for $\lambda \gg 1$. Further, we discuss the existence of a double S-shaped bifurcation curve when $n = 1$, $\Omega = (0, 1)$, and $f(s) = e^{\frac{\beta s}{\beta + s}}$ with $\beta \gg 1$.

Almost Linear Volterra Integro-differential Equations and Integral Equations
 Muhammad Islam
University of Dayton
 muhammad.islam@notes.udayton.edu
 Coauthors: Youssef Raffoul

Fixed point theorem of Krasnoselski is employed to study the existence of bounded continuous solutions of certain type of Volterra equations. These equations are studied under a set of assumptions. The equations will be called almost linear when these assumptions hold.

Generalized Diamond-Alpha Opial Dynamic Inequalities

Billur Kaymakcalan

Georgia Southern University

billur@georgiasouthern.edu

We begin by introducing a survey of Dynamic Opial Inequalities. Recent results of Opial type inequalities involving diamond-alpha derivatives and integrals are also given along with some applications.

Spectral properties of the matrix AKNS system and Schrödinger operators with complex potentials

Martin Klaus

Department of Mathematics, Virginia Tech, Blacksburg, VA 24061

klaus@math.vt.edu

We discuss the spectrum of the matrix AKNS systems (of which the two- component Zakharov-Shabat system is a special case) and establish bounds on the location of eigenvalues. In particular, we identify properties of the system that guarantee that the eigenvalues all lie on the imaginary axis. These results have implications for the spectrum of certain Schrödinger operators with complex potentials. This includes the class of PT-invariant (parity-time reversal-invariant), supersymmetric Hamiltonians.

A Multiplicity Result for a class of infinite positive problems

EunKyung Ko

Mississippi State University

ek94@msstate.edu

Coauthors: EunKyung Lee and R.Shivaji

We study positive solutions to the singular boundary value problem

$$\begin{cases} -\Delta_p u = \lambda \frac{f(u)}{u^\beta} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $p > 1, \lambda > 0, \beta \in (0, 1)$ and Ω is a bounded domain in $\mathbb{R}^N, N \geq 1$. Here $f: [0, \infty) \rightarrow (0, \infty)$ is a continuous nondecreasing function such that $\lim_{u \rightarrow \infty} \frac{f(u)}{u^{\beta+p-1}} = 0$. We establish the existence of multiple positive solutions for certain range of λ when f satisfies certain additional assumptions. A simple model that will satisfy our hypotheses is $f(u) = e^{\frac{\alpha u}{\alpha+u}}$ for $\alpha \gg 1$. We also extend our results to classes of systems when the nonlinearities satisfy a combined sublinear

condition at infinity. We prove our results by the method of sub-super solutions.

A NEW DIFFERENCE SCHEME FOR HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

Mehmet Emir Koksal

Halic University

emirkoksal@halic.edu.tr

Abstract Cauchy problem with the depending operator $A(t)$ (with domain $D(A(t))$) in a Hilbert space is considered. A new second-order accurate absolutely stable difference scheme generated by integer powers of $A(t)$ for approximately solving this problem is developed. The stability estimates for the solution of this difference scheme are established. Theoretical results are supported for solving one-dimensional hyperbolic partial differential equation with dirichlet boundary conditions.

Computer Use in a Mathematical Physics World

Kevin Kozlowski

University of Alabama at Birmingham

kevinkozlowski60@gmail.com

The purpose of the talk is to discuss computer programming issues that arise when creating programs to graph Mathematical and Mathematical Physics formulas and equations, with an end focus on Zero Range Potential Physics formulas. The talk will also demonstrate the benefits of using numerical methods in understanding Mathematical and Mathematical Physics formulas.

Well-posedness and long time behavior of finite energy solutions corresponding to 2-D nonlinear dynamic elasticity.

Irena Lasiecka
University of Virginia
il2v@unix.mail.virginia.edu

We shall discuss *finite energy* solutions corresponding to 2-D nonlinear systems of dynamic elasticity. Of particular interest to this talk are evolutions with "supercritical " (with respect to Sobolev's embeddings) nonlinearities. These include supercritical semilinear wave equations, von Karman evolutions, Kirchhoff Bousinesq equations and other non-linear plate equations. This class of models, while motivated by a multitude of physical applications, does not seem to have well established mathematical methodology for dealing with the issues such as existence, uniqueness and stability (including long time behavior) of solutions posed in physically significant *finite energy* space. The issue at hand is well recognized dichotomy between global existence and uniqueness. Existence for weak (finite energy) solutions is known, while uniqueness holds for strong solutions that are defined only locally.

In this talk we shall single out several classes of models for which rather general techniques have been recently developed. (see Ref [1-3] and references therein). These techniques lead -ultimately- to the resolution of the dichotomy and also provide a treatment for the study of long time behavior (global attractors) for the resulting dynamics. The results presented will supply positive answers to several open questions raised in the context of "supercritical" dynamic elasticity.

The methods discussed are based on tools such as : harmonic analysis, compensated compactness and logarithmic control of Sobolev's embeddings.

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- [2] G. Sell and Y. You. *Dynamics of Evolutionary Equations*. Springer Verlag, 2002
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Uniqueness and stability of determining coefficients of an inverse hyperbolic problem

Shitao Liu
University of Virginia
sl3fa@virginia.edu

We consider a second-order hyperbolic equation on an open bounded domain Ω in \mathbb{R}^n , with C^2 -boundary $\Gamma = \partial\Omega = \overline{\Gamma_0 \cup \Gamma_1}$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, subject to Neumann boundary conditions on the entire boundary Γ . $w_{tt}(x, t) - \Delta w(x, t) - q(x)w(x, t) = 0$ in $\Omega \times [0, T]$ and $\frac{\partial w}{\partial \nu}(x, t) = 0$ on $\Gamma \times [0, T]$ with given $w(\cdot, \frac{T}{2})$ and $w_t(\cdot, \frac{T}{2})$. We consider an inverse problem of determining $q(x)$, $x \in \Omega$, from Dirichlet data on the subboundary Γ_1 . Under suitable conditions on Γ_0 , Γ_1 and $T > 0$, we prove the uniqueness and the stability of $q(x)$. The proof is based on the Carleman estimate.

Strong Stability of a Nonlinear Fluid-Structure Interaction Model

Yongjin Lu

Department of Mathematics, University of Virginia

yl7m@virginia.edu

Coauthors: Irena Lasiecka

We consider a 2D fluid-structure interaction coupling the Navier-Stokes equations with a dynamic system of elasticity. In the model, an elastic solid is fully immersed in a fluid and the interaction is realized via an interface, the boundary of the elastic body. In the previous papers ([1],[2]), the authors proved the global-in-time existence and the regularity of weak solutions of the model. The goal of this paper is to establish the strong stability of the system for initial data in the natural energy space.

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Generalized Eigenproblem and Nonlinear Elliptic Equations

Nsoki Mavinga

University of Rochester, NY

mavinga@math.rochester.edu

We are concerned with the solvability of nonlinear second order elliptic partial differential equations with nonlinear boundary conditions. We study the generalized Steklov-Robin eigenproblem with (singular) weights and prove the existence of solutions for nonlinear problems when the nonlinearities interact, in some sense, with the generalized spectrum. The proofs are based on variational methods and a priori estimates.

A meshless method of lines for numerical solution of the family of generalized fifth-order Korteweg-de Vries equations

Elham Negahdary

University of Dayton

Elham.Negahdary@gmail.com

Coauthors: Muhammad Usman

In this paper we present a numerical solution of a family of generalized fifth-order Korteweg-de Vries equations using a meshless method of lines. This method uses radial basis functions for spatial derivatives and Runge-Kutta method as a time integrator. This method exhibits high accuracy as seen from the comparison with the exact solutions.

Spectral asymptotics of fractal Laplacians defined by iterated function systems with overlaps

Sze-Man Ngai

Georgia Southern University

smngai@georgiasouthern.edu

Coauthors: Jie Chen

We study the distribution of eigenvalues of some Laplacians defined by fractal measures. We focus on some well-known measures defined by iterated function systems with overlaps.

The Spectral Minimum of the Discrete Displacement Model

Roger Nichols, Jr.

University of Alabama at Birmingham

rnich_35180@yahoo.com

Coauthors: Gunter Stolz

We consider a discrete displacement model h_ω on $\ell^2(\mathbb{Z}^d)$ indexed by a displacement configuration $\omega = \{\omega_k\}_{k \in \mathbb{Z}^d}$. We determine a configuration ω^* with the property that h_{ω^*} has the lowest possible ground and highest possible ceiling state energies. We show that for the random displacement model, i.e. for ω_k i.i.d. random vectors, the configuration ω^* allows to calculate the minimum and maximum of the almost-sure spectrum. Using this result and positivity properties of generalized eigenfunctions, we show the almost-sure spectrum Σ_λ of the one-dimensional Bernoulli displacement model $h_\omega(\lambda)$, where $\lambda \in \mathbb{R} \setminus \{0\}$ is a fixed coupling constant, contains a gap for every $\lambda \neq 0$. We explicitly calculate Σ_λ for $0 < |\lambda| \leq 2$.

Bounded Solutions of Nonlinear Parabolic Equations in Unbounded Domains

M. N. Nkashama

University of Alabama at Birmingham

nkashama@math.uab.edu

Coauthors: N. Mavinga (University of Rochester, NY)

We are concerned with existence of bounded solutions for nonlinear second order parabolic equations on a domain that is unbounded in space-time with (possibly) a compact space-boundary. Exterior space-domains as well as the entire space are considered. No decay at infinity in the space variable is required. Using a combination of *a priori* estimates, comparison techniques, interpolation inequalities, approximation and imbedding of function spaces, we obtain existence results for bounded solutions existing for all times.

CHARACTERIZATION AND DISPERSIVE ORDERING OF THE CAUCHY, GAUSS AND LOGISTIC LAWS

Broderick O Oluyede

Georgia Southern University

boluyede@georgiasouthern.edu

In this talk, I present some results on the characterization and dispersive ordering of the general Cauchy, logistic and normal laws. The characterization of the Cauchy law is accomplished via a convex function of a symmetric random variable, as well as a differential equation involving the characteristic function. Results on the characterization of the logistic distribution shed further light into its application in a wide variety of areas including the analysis of quantal response and bioassay data, as well economic and demographic data. These results lead to necessary and sufficient conditions for the stochastic and dispersive ordering of the corresponding absolute random variables.

A model for the release of sterile mosquitoes

Seth F. Oppenheimer

Mississippi State University

seth@math.msstate.edu

Coauthors: with Renee Fister and Maeve Lewis McCarthy of Murray State University.

We construct a model for the release of sterile mosquitoes into a population of wild mosquitoes. The model assumes a Holling-II type dynamics as described by Jai Li in his July 2008 paper in *J. Bio Dynamics*. We then consider the problem where we allow diffusion. The result

is a pair of nonlinear parabolic partial differential equations. In this talk we will present the model and outline our proof for the case of Robin boundary conditions that the problem is well posed and positive nonnegative data yields nonnegative solutions. Our approach is via semigroups.

Existence of Multiple Positive Solutions to Some Semipositone Systems

Sarah Raynor

Wake Forest University

raynorsg@wfu.edu

Coauthors: Maya Chhetri and Stephen Robinson

In this paper we use the method of upper and lower solutions combined with degree theoretic techniques to prove the existence of multiple positive solutions to some semipositone, superlinear elliptic systems with Dirichlet boundary conditions, under suitable conditions on the nonlinearities. Our techniques apply generally to subcritical, superlinear problems with a certain concave-convex shape to their nonlinearity.

Reflection principles supported on a finite set

Ricardo Restrepo

Georgia Institute of Technology

restrepo@math.gatech.edu

We investigate the existence of reflection formulas supported on a finite set. It is found that for solutions of the *Laplace* and *Helmholtz* equation there are no finitely supported reflection principles unless the support is a single point. This confirms that in order to construct a reflection formula that is not ‘point to point’, it is necessary to consider a continuous support. For solutions of the wave equation $\partial^2 u / \partial x \partial y = 0$, there exist finitely supported reflection principles that can be constructed explicitly. For solutions of the telegraph equation $\partial^2 u / \partial x \partial y + \lambda^2 u = 0$, we show that if a reflection principle is supported on less than five points then it is necessarily a point to point reflection principle.

Fredholm Alternative Results for the Fucik Spectrum

Stephen Robinson

Wake Forest University

sbr@wfu.edu

Coauthors: Pavel Drabek

We provide solvability conditions for the boundary value problem

$$\begin{aligned}u'' + \alpha u^+ - \beta u^- &= f \text{ in } (0, \pi), \\u(0) = 0 &= u(\pi),\end{aligned}$$

where (α, β) is assumed to lie on a Fucik curve, and $f \in L_2(0, \pi)$ satisfies some variant of the standard orthogonality condition. The proofs rely on variational arguments.

An Ecological Model with Grazing and Constant Yield Harvesting

Sarath Sasi

Department of Mathematics and Statistics, Mississippi State University

ss885@msstate.edu

Coauthors: Ryan Causey and R. Shivaji

We study positive solutions to the steady state reaction diffusion equation with Dirichlet boundary condition of the form:

$$\begin{aligned}-\Delta u &= au - bu^2 - cu^p/(1 + u^p) - K, \quad x \in \Omega \\u &= 0, \quad x \in \partial\Omega.\end{aligned}$$

Here $\Delta u = \text{div}(\nabla u)$ is the Laplacian of u , a, b, c, p, K are positive constants with $p \geq 2$ and Ω is a smooth bounded region with $\partial\Omega$ in C^2 . This model describes the steady states of a logistic growth model with grazing and constant yield harvesting. It also describes the dynamics of the fish population with natural predation and constant yield harvesting. We study the existence of positive solutions to this model. We prove our results by the method of sub-super solutions.

Discontinuous Galerkin Solution of the Nonlinear Tricomi Equation

Adrian Sescu

University of Toledo

adrian.sescu@utoledo.edu

Coauthors: Abdollah A. Afjeh and Carmen Sescu

The Discontinuous Galerkin method applied to conservation laws has received considerable attention in the past years. It combines two key ideas characterizing finite element and finite volume methods, giving the high-order accuracy, the flexibility of handling complicated geometries, and the easy to treat boundary conditions. We apply a Discontinuous Galerkin method to the nonlinear Tricomi equation written in hyperbolic conservation law form; Legendre polynomials are used as the test functions. For the upper boundary, where the acoustic pressure field should match the geometrical acoustics approximation, a new mixed boundary condition is used. The analytical solution to the linear Tricomi equation is used to validate the Discontinuous Galerkin solution.

Modeling and analysis of system of differential equations for an Electrical machine

Kumud Singh-Altmyer

University of Arkansas Little Rock

kumud.altmyer@yahoo.com

Coauthors: Kamran Iqbal University of Arkansas Little Rock

Abstract: This work deals with the study of system of equations arising from a dynamic model of an electrical machine. These set of differential equations represent relationship between the voltages applied and currents, fluxes produced in the electrical system. In general this phenomenon happens when an electromagnetic field is created in an electric machine which could be a transformer or an Induction machine. The structure of these two machines could be similar though behavior in some sense may not be same. In present case we consider the machine windings for homogeneous or non-homogeneous medium with Dirichlet and Mixed Boundary conditions. A comparison of exact solutions to numerical solutions is established. The field oriented control analysis is done by using Fuzzy logic control method and compared with old methodology of PI control method. The Fuzzy control method provides stability and robustness of an electrical machine.

An example is under consideration that will depict how equations and solutions behave differently in different media/materials with respect to power and torque/speed control. The software used are FEMLAB, MATLAB.

Key Words: AC machine, Field oriented control, Closed loop control

Stability of traveling waves in nonlinear damped hyperbolic equations
Zachariah Sinkala
Department of Mathematical Sciences , Middle Tennessee state University,
Box34, Murfreesboro Tennessee 37132
zsinkala@mtsu.edu

We consider the nonlinear damped hyperbolic equation

$$\epsilon u_{tt} + u_t = \partial_x (u^m \partial_x u) + F(u), \quad x \in R, \quad t \geq 0, \quad u \in R$$

where $\epsilon > 0$, not necessarily small parameter. We assume that

$$F(0) = F(\alpha) = F(1) = 0, \quad F'(0) < 0, \quad F'(1) < 0 < F'(\alpha).$$

Under these assumptions, our equation has a continuous family of monotone traveling waves indexed by the speed parameter $c \geq c^*$. Using energy estimates and additional assumption on the nonlinearity, we obtain global stability using a suitable version of the hyperbolic maximum principle. Finally, in the critical case $c = c^*$, we use self-similar variables to compute the exact asymptotic behavior of the perturbations as $t \rightarrow +\infty$. In particular, letting $\epsilon = 0$, we recover stability results for the traveling waves of the corresponding parabolic equation.

Integral Invariants of a Nonlinear Nonholonomic Dynamical System in Poincare Parameters.

Muhammad Usman

University of Dayton

Muhammad.Usman@notes.udayton.edu

Coauthors: Naseer Ahmed

We formulate general equations of motion for mechanical systems subject to nonlinear nonholonomic constraints, that do not involve Lagrangian undetermined multipliers. These equations are written in a canonical form called the Poincare-Hamilton equations, and study a version of corresponding Poincare-Cartan integral invariant which are derived by means of a type of asynchronous variation of the Poincare variables of the problem that involve the variation of the time. As a consequence, it is shown that the invariance of a certain line integral under the motion of a mechanical system of the type considered characterizes the Poincare-Hamilton equations as underlying equations of the motion.

Bifurcation Analysis of a Kaldor-Kalecki Model of Business Cycle with Time Delay

Liancheng Wang
Kennesaw State University
lwang5@kennesaw.edu
Coauthors: Xiaoqin Wu

In this research, we investigate a Kaldor-Kalecki model of business cycle with delay in both the gross product and the capital stock. Stability analysis for the equilibrium point is carried out. We show that Hopf bifurcation occurs and periodic solutions emerge as the delay crosses some critical values. By deriving the normal forms for the system, the direction of the Hopf bifurcation and the stability of the bifurcating periodic solutions are established. Examples are presented to confirm our results.

A mathematical model on the dynamics of phytoplankton and zooplankton prey-predator system

Xubo Wang
Macon State College
lily.wang@maconstate.edu
Coauthors: Azmy S. Ackleh and Keng Deng, Department of Mathematics, University of Louisiana at Lafayette

A model that describes the dynamics of phytoplankton and zooplankton prey-predator system within the context of phytoplankton aggregation is discussed. Existence-uniqueness results of the solution are established via a comparison principle and the upper-lower solution technique.

Modeling the biomass at nearly pristine coral reefs

Howard (Howie) Weiss
Georgia Institute of Technology
weiss@math.gatech.edu

Until recently, the only examples of inverted biomass pyramids have been in freshwater and marine planktonic communities. In 2002 and 2008 investigators documented inverted biomass pyramids for nearly pristine coral reef ecosystems within the NW Hawaiian islands and the Line Islands, where apex predator abundance comprises up to 85% of the fish biomass. We build a new refuge based ODE predator-prey model to study the fish biomass structure at coral reefs and investigate the effect of fishing on biomass pyramids. Utilizing realistic life history parameters of coral reef fish, our model exhibits a stable

inverted biomass pyramid. Since the predators and prey are not well mixed, our model does not incorporate homogeneous mixing and the inverted biomass pyramid is a consequence of the refuge. Understanding predator-prey dynamics in nearly pristine conditions provides a more realistic historical framework for comparison with fished reefs. Finally, we show that fishing transforms the inverted biomass pyramid to be bottom heavy.

Collinear Central Configurations in the n -body Problem with General Homogeneous Potential

Zhifu Xie

Virginia State University

zxie@vsu.edu

Coauthors: Mervin Woodlin

In this paper we investigate the central configurations of collinear n -body problem given by the general law of attraction of the form $f(r) = \frac{1}{r^\alpha}$. A method involving analysis skills of some elementary algebra and calculus is presented to study the central configurations in the collinear n -body problem. It is well known that for given n positive masses there are precisely $n!/2$ collinear central configurations for Newton's law of gravitation of $\alpha = 2$ (F. R. Moulton, The straight line solutions of the problem of N bodies, Ann. Math., II. Ser. 12, 1910) and Smale reconfirmed the result by a variational method in 1970. However, it is not true that there is always a position which causes a central configuration for any given ordered particles with some positive masses, and there may exist more than one position which make it central for some $\alpha < 0$. We give a generalization of Moulton's theorem for collinear n -body problem for all $\alpha > 0$. Examples that Moulton's theorem does not work are also provided.

Positive Solutions for Infinite Semipositone Problems with Falling Zeros

Jinglong Ye

Center for Computational Sciences, HPC, Mississippi State University

jy79@msstate.edu

Coauthors: EunKyoung Lee and R. Shivaji

We consider the positive solutions to the singular problem

$$\begin{cases} -\Delta u = au - f(u) - c/u^\alpha & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $0 < \alpha < 1$, $a > 0$ and $c > 0$ are constants, Ω is a bounded domain with smooth boundary and $f: [0, \infty) \rightarrow \mathbb{R}$ is a continuous function. We assume that there exist $M > 0$, $A > 0$, $p > 1$ such that $au - M \leq f(u) \leq Au^p$, for all $u \in [0, \infty)$. A simple example of f satisfying these assumptions is $f(u) = u^p$ for any $p > 1$. We use the method of sub-supersolutions to prove the existence of positive solution when $a > \frac{2\lambda}{1+\alpha}$ and c is small. Here λ is the first eigenvalue of operator $-\Delta$ with Dirichlet boundary conditions. We also extend our result to classes of infinite semipositone systems.

Some Eigenvalue inequalities for Klein-Gordon operators.

Selma Yildirim-Yolcu

Georgia Institute of Technology and Georgia College&State University

selma@math.gatech.edu

Coauthors: Evans Harrell.

In this talk, we present several universal inequalities for the eigenvalues for the pseudodifferential operators $H_{m,\Omega}$ associated by the prescriptions of quantum mechanics to the Klein-Gordon Hamiltonian when restricted to a compact domain Ω in \mathbb{R}^d .

Maxwell-Systems with Nonlinear Polarization Involving Probability Distributions

Younis Zaidan

University of Wisconsin-Fox Valley

younis.zaidan@uwc.edu

We consider the propagation of high frequency electromagnetic pulses in complex materials with nonlinear polarization. The physical problem is modeled by Maxwell's equations in variational form, and well-posedness results are established with respect to probability distributions on the polarization parameters in a Prohorov metric sense.

Asymptotic behaviors of solutions to a class of quasilinear elliptic equations with large diffusion

Chunshan Zhao

Georgia Southern University

czhao@GeorgiaSouthern.edu

I will talk about asymptotic behaviors of solutions to a class of quasilinear elliptic equations as diffusion coefficient goes to infinity. both Neuman problem and Dirichlet problem of the elliptic equations will be included.

Computation of the H_∞ -Control Attenuation Parameter Using Sensitivity Analysis

Lizette Zietsman

Virginia Tech

lzietsma@vt.edu

In order to design a meaningful control law using H_∞ control, it is necessary to know the optimal attenuation, denoted by θ^* . The results by Doyle *et al.* (1989) give a necessary and sufficient condition for the existence of an admissible controller such that the H_∞ -norm of the transfer function is less than a prescribed value, say $1/\theta$. This leads to an iterative algorithm to determine θ^* . At each step, one has to verify the positive semi-definiteness and the stabilizing properties of the solutions to two algebraic Riccati equations. In addition, one must check if a coupling condition is satisfied. In order to verify this second condition, it is necessary to determine if a certain coupling matrix is positive definite.

In this paper we investigate the role that the sensitivity of the eigenvalues of the coupling matrix and the sensitivity of the Riccati solutions play in determining the optimal attenuation, θ^* . We present numerical results that illustrate an algorithm where the sensitivity of an eigenvalue is used to determine θ^* . In this computation, we can also verify the conditioning of the associated Riccati equations. The numerical experimentation indicates that the order of convergence of this algorithm is linear.